Ponzirroni: An empirical test of beating the market

Dean P. Foster and Robert A. Stine
Department of Statistics, The Wharton School
University of Pennsylvania, Philadelphia PA
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Story

- Someone describes their new investment “scheme.”
- It has a wonderful track record.
- But, is it real or is it delusional?
- (Our research was inspired by the existence of hedge funds.)

Our question: What does it take for a historical track record to have statistically significantly beaten the market?
Summary of talk

• Discuss both toy examples and real examples of excess returns
  
  – Toy examples: generate excess returns using silly methods
  
  – Real examples: individual stocks, and factor level

• Rules for excess returns
  
  – t-test (with Bennett correct)
  
  – t-test (with returns approximation
  
  – martingale test

• Toys show that tests are tight

• Are they too tight for anything to be significant?
  
  – business cycles ... probably no
  
  – individual stock ... quite possible
Bonferroni

TOY: Friend tells you they found a stock that has spectacular returns over the past 5 years. Do you buy?

- Stocks are incredibly independent of each other.
  - Liang Wang showed about 50% (or more) idiosyncratic risk
  - So each stock is its only “independent” series

- Naively we expect one stock to have gone up by at least 10 times in the past 5 years. (assuming IID normal returns)

- Bonferroni is the standard way to correct for this.
Peso problem

TOY: Friend tells you of a scheme that has beaten the market every month for the past 2 years. 24 victories in a row. Wow!

- Mean can still be worse than market
  - most of the time, it beats the market by 1%
  - only 1/50 months does it lose to the market by 100%
  - Mean is worse than market, median is better

- Finance is not the place to use robust statistics!

- Must protect from unobserved downside.

Leverage

TOY: Friend describes scheme that has higher “mean” than the market. Scheme has worked for more than 80 years. No tricks with medians here.

• Eg: Leveraging the market by any factor greater than 1 has a higher average return than the market over last 80 years

• More impressive: Leveraging the market by factor of 2 has higher log returns than the market

• Most people can’t stomach this much risk

• Need to correct for risk
The Ponzirroni test

The Ponzirroni test is

1. Compute the intercept $\alpha$ in the CAPM regression of the return series, namely

$$(r_t - r_t^f) = \alpha + \beta(r_t^m - r_t^f) + \text{noise}$$

where

- $r_t$ = returns for process under consideration
- $r_t^f$ = risk free returns
- $r_t^m$ = market returns

2. Use Bennett’s bound to compute a p-value for this $\alpha$

3. Multiply this p-value number of tests tried

This is the Ponzirroni p-value.

- If it is significant, be impressed.
- Be very impressed. And call me.
Bennett bound?

Bennett's bound for tail probability (1962)

- Bounded independent summands $B_i$, $\sup |B_i| \leq M$.

- $E B_i = 0$, $\sum_i \Var(B_i) = 1$.

$$P\left( \sum B_i \geq \tau \right) \leq \exp \left( \frac{\tau}{M} - \left( \frac{\tau}{M} + \frac{1}{M^2} \right) \log(1 + M \tau) \right)$$
Easier bound: (Maurer 2000)

\[ P(\sum B_i \geq \tau) \leq \exp \left( \frac{-\tau^2}{2n(\sigma^2 + M^2)} \right) \]

- Bennett bound is asymptotically efficient if \( n \) gets large.

- Maurer gets the variance wrong in the limit.
Properties of the Ponzirroni test

• Protects against all three toy examples

• Doesn’t protect against
  – trading costs
  – bid-ask bounce
  – tax issues
  – etc

• Will be close to usual t-test under “most” situation
  – i.e. lots of data with small returns
  – Hedge funds can trick the t-test. Must use Ponzirroni in this case.
Teaching PonziRoni

I’ve taught these concepts to MBA’s and mathematics undergraduates. Some changes help make it more understandable:

- Talk about ponzi schemes.

- Talk about Mathnet’s version of a Ponzi scheme
  - This gets the idea of doubling.
  - And the idea of selection bias.

- Talk about the “rule of 3” instead of Bennett.

- Have them do simulations of the various methods rather than probability calculations
A simple business cycle model

How big would market swings have to be to pass a Ponzirroni test?

Model assumptions:

- We know bull/bear indicator
- Half bull, half bear
- Cycles ever $t$ months (on average)
- Bull generates $r$ excess return per cycle (on average)
- Observe for 50 years
- Test $t = 1$ month, 2 month, 4 month, 8 month, etc to 10 years.
- 7 tests in all
Applying Ponzirroni to business cycles

(Cycles length = $t$ months, Bull generates $r$ excess return per cycle)

- With 50 years of data, to be significant $r > t \times .8\%$.
  - $t = 5$ years means $r > 50\%$
  - $t = 1$ year means $r > 10\%$
  - Identified bull periods must produce 17% per year
  - Identified bear periods must lose -3% per year
Applying Ponzirroni to individual stocks

- Suppose the sentence “blah blah blah IBM blah blah disaster blah blah blah.” Did not bode well for IBM’s return the next day.

- Called bag of words model.

- My graduate student Liang Wang did this
  - Looked at one year of data.
  - searched 1000 words.
  - Reuters news service.
  - (Aside: Primary goal was to forecast volatility)

- Generalized Estimating Equations is a good approximation to the Ponzirroni test in this case.
Applying Ponzirroni to individual stocks

- What change in return is detectable?

- SE’s are typically 10 basis points due to the rarity of “disaster” occurring.

- If a “buy” signal occurred about 1/2 the time, the SE would be less than 1 basis point.

- BTW, nothing was found to be significant.
Fama/French factors

- Value vs. Growth
  - High book / market = value
  - Low book / market = glamor?
  - CLAIM: value is better
  - CLAIM: small stocks are better

- Psychological impact of claim
  - CLAIM: B/M portfolios are independent of market (i.e. beta is zero)
  - CLAIM: Generate higher returns than market
  - With these two claims, you can double your returns over the market
NOTE TO BOB: It shows returns for a value portfolio and a glamor portfolio... I did not get a chance to see this.

It considers these returns “obviously” better. No statistics are provided, but 20 years of data are included.

If you want to peek at the data—it is at

gosset:~foster/value_vs_growth_FF.jmp
Ponzirroni analysis

• I’ll be using data from 2004 review article by Chan and Lakonishok

• Show data

• Looks impressive doesn’t it?

• Paper’s definition of “value” and “glamor” not given. But it appears to have at least 4 parameters. So we will use 4 for our multiplicity.

• t-statistic of alpha is insignificant
Conclusions

• Ponzirroni tests are tight: any weakening of methodology will allow silly schemes to look impressive

• Need lots of data
  – Economically significant differences can’t always be supported statistically.
  – Looking at whole market is very difficult. Returns must be spectacular.
  – Looking at patterns of individual stocks is much more accurate.

• Fun to teach in class.